Qubit rotation and Berry phase

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Abstract

A quantized fermion can be represented by a scalar particle encircling a magnetic flux line. It has the spinor structure which can be constructed from quantum gates and qubits. We have studied here the role of Berry phase in removing dynamical phase during one qubit rotation of a quantized fermion. The entanglement of two qubit inserting spin-echo to one of them results the change of Berry phase that can be considered as a measure of entanglement. Some effort is given to study the effect of noise on the Berry phase of spinor and their entangled states.

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1 Introduction

Entanglement is one of the basic aspects of quantum mechanics.It was known long ago that quantum mechanics exhibits very peculiar correlations between two physically distant parts of the total system. Afterwards, the discovery of Bell's inequality (BI) [1] showed that BI can be violated by quantum mechanics but have to be satisfied by all local realistic theories. The violation of BI demonstrates the presence of entanglement [2].

It is well known that the geometrical phase, such as the Berry phase [3] play an important role in quantum mechanics. The geometric origin of this phase is given by the holonomy of the line bundle of the states where the phase emerges from the integral of the connection (or curvature) of the bundle over the parameter space [4].In recent years geometric phase in a single particle system has been studied very well, both theoretically and experimentally. The effect of Berry phase on entangled quantum system is less known. But there is an interest to combine both these quantum phenomenon[5].

It is natural to think that there is an inherent connection of these two important quantum phenomena namely,entanglement and Berry phase with quantization procedure. The aim of this paper is to explore that connection.

Bell's inequality theorem may be interpreted as incompatibility of requirement of locality with the statistical predictions of quantum mechanics. So to study the Bell state, the role of *local* spatial observations, apart from spin correlations, should also be taken into considerations[6]. Since the Berry phase is acquired by the space-time wave function when a particle traverses a closed path and in the realm of quantum field theory it is associated with a gauge field, the role of Berry phase becomes relevant in an entangled state. Indeed, chiral anomaly is a purely quantum effect which arises due to the short distance singularity and it is expected that the influence of the Berry phase on an entangled state is somehow linked up with that of the local observations of spins. This suggests that to have a comprehensive view of the quantum mechanical correlation of two spin 1/2 particles in an entangled state we should take into account the role of the Berry phase related to a spinor. In this note we shall study the formation of spinor by the operation of quantum gates on qubits. We will investigate the role of Berry phase in the one-qubit rotation in the presence of the circulating magnetic field and entangled state of two qubit. Further we here also show some interest on the appearance of noise in the topological phase of the pure and entangled state.

2 Quantization of a Fermi field and Berry Phase

The quantization of a Fermi field can be achieved when we introduce an anisotropy in the internal space through the introduction of a direction vector as an internal variable[7]. The opposite orientations of the direction vector correspond to particle and antiparticle. To be equivalent to the Feynman path integral we have to take into

account complexified space-time when the coordinate is given by $Z_\mu = x_\mu + i\xi_\mu$ where ξ_μ corresponds to the direction vector attached to the space-time point x_μ. Then the field function will be of the form $\phi(x_\mu, \xi_\mu)$ which may describe a particle moving in an anisotropic space. If χ be the angle to specify the rotational orientation around the direction vector ξ_μ, then the wave function will depend on another quantity μ apart from the coordinates r, θ, ϕ where μ is the eigenvalue of the operator $i\frac{\partial}{\partial\chi}$. Here μ corresponds to the measure of anisotropy and behaves like the strength of a magnetic monopole.Indeed in this space the angular momentum is given by

$$\vec{J} = \vec{r} \times \vec{p} - \mu\hat{r} \tag{1}$$

with $\mu = 0, \pm 1/2, \pm 1...$ This corresponds to the motion of a charged particle in the field of a magnetic monopole.

The spherical harmonics incorporating the term μ may be written as [8]

$$Y_l^{m,\mu} = (1+x)^{-(m-\mu)/2}.(1+x)^{-(m+\mu)/2} \times \frac{d^{l-m}}{d^{l-m}x}[(1+x)^{l-\mu}.(1-x)^{l+\mu}]e^{im\phi}e^{i\mu\chi} \tag{2}$$

with $x = cos\theta$.

Since the chirality is associated with the angle χ denoting the rotational orientation around the *direction vector* ξ_μ, the variation of the angle χ i,e will correspond to the change in chirality. In spherical harmonics given by eqn.(2) the angular part associated with the angle χ is given by $e^{-i\mu\chi}$. Thus when χ is changed to $\chi + \delta\chi$, we have [9]

$$i\frac{\partial}{\partial(\chi+\delta\chi)}e^{-i\mu\chi} = i\frac{\partial}{\partial(\chi+\delta\chi)}e^{-i\mu(\chi+\delta\chi)}e^{i\mu\delta\chi} \tag{3}$$

which implies that the wave function will acquire the extra phase $e^{i\mu\delta\chi}$ due to infinitesimal change of the angle χ to $\chi + \delta\chi$. When the angle χ is changed over the closed path $0 \leq \chi \leq 2\pi$, for one complete rotation, the wave function will acquire the phase

$$exp[i\mu\int_0^{2\pi}\delta\chi] = e^{2i\pi\mu} \tag{4}$$

which represents the Berry phase. Indeed in this formalism, a fermion is depicted as a scalar particle moving in the field of a magnetic monopole and $\mu = 1/2$ corresponds to one flux quantum. When a scalar field(particle) traverses a closed path with one flux quantum enclosed, we have the phase $e^{i\pi}$ which suggests that the system represents a fermion.

In general this Berry phase is the solid angle subtended by the particle that can be seen by considering the two component spinor structure of quantized spin 1/2 particle. For the specific case of $l = 1/2, |m| = |\mu| = 1/2$ from the spherical harmonics, we can construct the instantaneous eigenstates $|\uparrow, t>$ as

$$|\uparrow, t> = \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} {Y_{1/2}}^{1/2,-1/2} \\ {Y_{1/2}}^{-1/2,-1/2} \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2}\exp i(\phi-\chi)/2 \\ \sin\frac{\theta}{2}\exp -i(\phi+\chi)/2 \end{pmatrix} \tag{5}$$

that can be written in terms of qubits $|0> and|1>$ as follows

$$| \uparrow, t >= [\cos \frac{\theta}{2}|0> + \sin \frac{\theta}{2} e^{-i\phi}|1>] e^{i/2(\phi-\chi)} \tag{6}$$

In similar to the coherent state approach [10], the effective Lagrangian for this state becomes

$$L^{\uparrow}_{eff} =<\uparrow, t|\nabla_t| \uparrow, t>= -\frac{i}{2}(\dot{\chi} - \dot{\phi}\cos\theta) \tag{7}$$

The action integral over a closed path gives rise from the eqn.-7, the required geometrical phase of the single quantized spinor.

$$\gamma_{\uparrow} = i \int L^{\uparrow}_{eff} dt = i \oint A_{\uparrow}(\lambda) d\lambda = \frac{1}{2}(\oint d\chi - \cos\theta \oint d\phi) = \pi(1-\cos\theta) \tag{8}$$

This shows that for quantized spinor the Berry Phase is a solid angle subtended about the quantization axis. For $\theta = 0$ the minimum value of $\gamma_{\uparrow}$ is 0 and at $\theta = \pi$ maximum.

The conjugate spinor of equation-6

$$| \downarrow (t) >= (\sin \frac{\theta}{2}|0> + \cos \frac{\theta}{2} e^{i\phi}|1>) e^{-i/2(\phi-\chi)} \tag{9}$$

possesses the Berry phase over the closed path

$$\gamma_{\downarrow} = \pi(1+\cos\theta) \tag{10}$$

which is maximum for $\theta = 0$,and minimum $\gamma_{\downarrow} = 0$ for $\theta = \pi$. It can be verified that this Berry phase remains the same if we neglect the overall phase $e^{\pm i(\phi-\chi)/2}$ from the quantized spinors as in eqns. 6 and 9 respectively. Because we obtain the identical value of Berry phase $\gamma_{\uparrow\downarrow}$ in both the approach identifying the same solid angle subtended about the axis of anisotropy. It implies also that the variable θ plays the crucial role in visualizing the Berry phase.

From the view point of quantum computation we will now proceed to study the rotation of quantized spinor. A microscopic system such as an atom,a nuclear spin or a polarized photon can exist in arbitrary superposition of $\alpha|0> + \beta|1>$ where $|0>$ and $|1>$ represent the ground and excited state respectively. In other words any time dependent wave-function can be written as

$$\Psi(t) = C_0(t)|0> + C_1(t)|1> \tag{11}$$

Most general pure state of a single qubit can be sufficiently constructed using the two well known quantum gates - Hadamard gate(H) and Phase gate as follows

$$|0> - - - [H] - - - - \bullet^{2\theta} - - - [H] - - - - \bullet^{\pi/2+\phi} \longrightarrow \cos\theta|0> + \sin\theta e^{i\phi}|1> \tag{12}$$

The bracketed term of the quantized spinor(equation 6), can be written apart from the phase factor in terms of the two qubits and the Hadamard and phase gates as follows

$$|0> ---[H]---\bullet^{\theta}---[H]---\bullet^{\pi/2-\phi} \longrightarrow \cos\frac{\theta}{2}|0> + \sin\frac{\theta}{2}e^{-i\phi}|1> \quad (13)$$

Now we will focus on developing an understanding of the time evolution of a single qubit by a general Hamiltonian. Any 2×2 hermitian matrix can be written in terms of unit matrix and the three Pauli matrices.

$$H = \frac{\hbar}{2}(\Omega_0\mathbf{1} + \mathbf{\Omega}.\sigma) \quad (14)$$

where Ω_0 is the frequency of applied magnetic field and Ω called the Rabi frequency which describes the transition between the ground state $|0>$ and excited state $1>$, under the action of the resonant field.

According to Berman, Doolen et.al [11], in the presence of rotating magnetic field the time dependent wave-function eqn.11 are governed by unitary transformation

$$\Psi(t) = U(t)\Psi(0) = U(t)[C_0(0)|0> + C_1(0)|1>] = C_0(t)|0> + C_1(t)|1> \quad (15)$$

where the time dependent coefficients of qubits satisfy the following equation.

$$\begin{array}{rcl} C_0(t) & = & C_0(0)\cos\frac{\Omega t}{2} + iC_1(0)\sin\frac{\Omega t}{2} \\ C_1(t) & = & C_0(0)i\sin\frac{\Omega t}{2} + C_1(0)\cos\frac{\Omega t}{2} \end{array} \quad (16)$$

The characteristic time of this transition $t = \pi/\Omega$ is usually much longer than the period of precession, so that slow and fast variables lead to some Born-Oppenheimer approximation. This $t = \frac{\pi}{\Omega}$ is also considered as the duration of the external field. If initially spin is in the ground state at $(t = 0)$, for $C_0(0) = 1, C_1(0) = 0$, the time dependent coefficients become

$$C_0(t) = 0, \qquad C_1(t) = i \quad (17)$$

Thus a pulse of a resonating magnetic field with a duration π/Ω drives the system from the ground state to the excited state. Such a pulse π, conversely drives a spin from excited to ground state also.If we apply a pulse with different duration, we can drive the quantum system into a super-positional state, creating a rotation of one qubit.Also when $t = \pi/2\Omega$ the resonating magnetic field has a $\pi/2$ pulse driving the system as super-positional state of ground and excited state in equal weight.

It seems that this rotation in the presence of magnetic field, obviously will lead to the formation of Berry phase through the quantization procedure. We here consider that action of π pulse on our quantized spinor in eqn. 6 initially at the ground state $|0>$, resulting its transfer to the excited state $|1>$, only when

$$\begin{array}{l} C_0(t) = \cos\frac{\theta}{2}e^{i/2(\phi-\chi)} = 0 \\ C_1(t) = \sin\frac{\theta}{2}e^{-i/2(\phi+\chi)} = i \end{array} \quad (18)$$

which will be possible for

$$\theta = \pi \quad and \quad (\phi + \chi)/2 = -\pi/2$$

. This further indicate in connection with our previous analysis in eqn.8 that only $\gamma_\uparrow$ will be visible at $\theta = \pi$.

To rotate the spinor once, another π pulse is given in order to send the spinor from excited state back to ground one. Thus

$$C_0(0) = 0, C_1(0) = 1$$

is considered initially and the action of a π pulse to the quantized spinor result

$$\begin{array}{l} C_0(t) = \cos\frac{\theta}{2} e^{i/2(\phi-\chi)} = i \\ C_1(t) = \sin\frac{\theta}{2} e^{-i/2(\phi+\chi)} = 0 \end{array} \qquad (19)$$

which indicate the possibility of

$$\theta = 0 \qquad and \qquad (\phi - \chi)/2 = \pi/2$$

visualizing $\gamma_\downarrow$ only. From the above equivalence we can realize that this one qubit rotation by the application of two π-pulses in presence of the circularly polarized magnetic field, is equivalent to the rotation of a quantized particle over a closed path where the spinor changes from ground state ($|0>$) to excited state ($|1>$) again to ground state($|0>$) or the converse. Hence the net phase acquired in this case is $(\phi+\chi+\phi-\chi)/2 = -\pi/2+\pi/2 = 0$. This implies that the dynamical phase vanishes where in our picture the variable ϕ is its very source. On the other hand if $(\phi + \chi - \phi + \chi)/2 = \pi/2 - (-\pi/2)$, we have only the Berry phase $\chi = -\pi$.

Thus in this course of 'spin-echo' method,the dynamical phase of the quantized spinor can be removed if $(\phi + \chi) = -(\phi - \chi)$ is followed. For $\theta = 0$, the Berry phase(BP) for spin up state ($\gamma_\uparrow$) can be removed whereas for $\theta = \pi$ the BP for down spin ($\gamma_\downarrow$) vanishes.

3 Berry Phase in an Entangled state of two spin-1/2 particles

In the scheme of quantization of a Fermi field, the *direction vector* effectively represents a vortex which is equivalent to a magnetic flux line. Mathematically, μ is associated with this magnetic flux quantum. Since our Berry phase visualizing through μ, depends on the continuous values of θ, to study the behavior of the Berry phase factor, we take the resort of the μ-theorem [12].It implies that μ can take some continuous values where the fixed points of the Rg flow are the physical values of the monopole strength. We may take μ not to be a fixed value but dependent on a parameter λ and the function $\mu(\lambda)$ should satisfy:
1.μ is stationary at the fixed points $\mu^* = \mu(\lambda^*)$ of the RG flow i,e $\nabla\mu(\lambda^*) = 0$.

2.At the fixed points $\mu(\lambda^*)$
3.μ is decreasing along the infrared RG flow.i,e

$$L\frac{d\mu}{dL} \leq 0 \tag{20}$$

where L is a length scale. We can specify

$$L\frac{d\mu}{dL} = -a, a \geq 0 \tag{21}$$

Solving this we find

$$\mu = -alnL + c \tag{22}$$

where c is an arbitrary constant. Indeed neglecting the constant term c, in eqn.22, we can write

$$\mu = -aln|x - y| \tag{23}$$

when the two interacting particles are situated at the points x and y respectively. The effect of one on the other is depicted through this relation in eqn 23 and its role can be observed in the entanglement of two particles. We now will study the appearance of Berry phase in the entanglement of two identical spin 1/2 quantized particles. The antisymmetric Bell state of two spin 1/2 particles is

$$|\Psi >= \frac{1}{\sqrt{2}}(|\uparrow_n> |\downarrow_n> -|\uparrow_n> |\downarrow_n>) \tag{24}$$

If we use spinor depicted by eqn.13 in the above Bell state, we find after few mathematical steps that the entangled state $|\Psi >$ at a particular instant t is connected with the primary entangled state $|\Psi_0 >$ of qubits

$$|\Psi >= \cos\theta|\Psi_0 >= \frac{1}{2\pi}(\gamma_\uparrow - \gamma_\downarrow)|\Psi_0 > \tag{25}$$

by the difference of Berry phase factor. Here $|\Psi_0 >$ is considered as the initial entangled state constructed from the qubits $|0 >$ and $|1 >$.

$$|\Psi_0 >= 1/\sqrt{2}(|1 > |0 > -|0 > |1 >) \tag{26}$$

Hence as the two quantized spin 1/2 come close to each other, we have the Berry Phase of their entangled state

$$\gamma_{ent} = \oint < \psi|\nabla\psi > d\phi = \pi(1 + \cos 2\theta) \tag{27}$$

It seems that the entangled state after one rotation though acquire Berry phase but rotation of Bell state from symmetric Ψ_+ to antisymmetric Ψ_- state does not takes place. The difference of $\gamma_{\uparrow,\downarrow}$ in eqn.27 implies as if the net topological effect due to quantization disappears. The study of Ghrirardi et.al[13] regarding the non-entanglement of identical fermions is different from our study due to findings after second quantization.

With these view, two identical fermions are made quite different before entanglement, by introducing one qubit rotation to one of them through the circularly polarized magnetic field that plays the role of changing the direction of quantization axis. It is done in such a way that a resonance with the precession of the spin vector is formed. Significantly the rotation should possess the change of Entangled state from symmetric to antisymmetric state over the closed path. This rotation of the magnetic field effectively corresponds to the change in the direction of the magnetic flux line. The instantaneous eigenstate of the two conjugate spin operator in the direction of the $n(\vec{\theta/2}, t)$ expanded in the σ_z basis are our quantized spinors

$$|\uparrow(t)>= (\cos\frac{\theta}{2}|0> + \sin\frac{\theta}{2}e^{-i\phi}|1>)e^{i(\phi-\chi)/2} \tag{28}$$

$$|\downarrow(t)>= (\sin\frac{\theta}{2}|0> + \cos\frac{\theta}{2}e^{i\phi/2}|1>)e^{-i(\phi+\chi)/2} \tag{29}$$

For the time evolution from $t = 0$ to $t = \tau$ each eigen state will pick up a Berry phase apart from the dynamical phase [14].

$$|\uparrow; t = 0 >\rightarrow |\uparrow; t = \tau >= e^{i\gamma_\uparrow(\theta)}e^{i\tilde{\theta}_+}|\uparrow_n; t = 0> \tag{30}$$

$$|\downarrow; t = 0 >\rightarrow |\downarrow; t = \tau >= e^{i\gamma_\downarrow(\theta)}e^{i\tilde{\theta}_-}|\downarrow_n; t = 0> \tag{31}$$

where $\gamma_{\uparrow,\downarrow}$ denotes the Berry phase which is half of the solid angle swept out by the magnetic flux line and $\tilde{\theta}_\pm$ is the dynamical phase. The explicit values of these phase $\gamma_{\uparrow,\downarrow}$ are

$$\gamma_\uparrow(\theta) = \pi(1 - cos\theta) = 2\pi\mu_\uparrow \tag{32}$$

$$\gamma_\downarrow(\theta) = \pi(1 + cos\theta) = 2\pi\mu_\downarrow = 2\pi - \gamma_\uparrow \tag{33}$$

where we define $\mu_{\uparrow,\downarrow} = \frac{1}{2}(1 \mp \cos\theta)$ as the measure of anisotropy.

If we apply spin-echo method to one spinor before entanglement with other then the Berry phase is trapped in the entangled state, resulting the removal of dynamical phase. This benefits to change the entangle Bell state from antisymmetric to symmetric one at a particular position. The most general antisymmetric Bell state for two particles A and B situated at the points x and y becomes

$$|\Psi(t)>= (\alpha|\uparrow(t)>|\downarrow(t)> -\beta|\uparrow(t)>|\downarrow(t)> \tag{34}$$

where α and β are two complex coefficients,

With the idea of one qubit rotation of one fermion for a time interval τ the spinor comes to its orginal state acquiring only Berry phase and loosing the dynamical phase. The Bell state becomes

$$|\Psi(t = \tau)>= (e^{i\gamma_\uparrow}\alpha|\uparrow(t)>|\downarrow(t)> -e^{i\gamma_\downarrow}\beta|\uparrow(t)>|\downarrow(t)> \tag{35}$$

Neglecting the over all phase, we have the new form of the entangle state as

$$\Psi(t = \tau) = (\alpha|\uparrow(t)>|\downarrow(t)> -e^{2i\gamma_\uparrow}|\uparrow(t)>\beta|\downarrow(t)> \tag{36}$$

As we consider $\theta = \pi$ the Berry phase is removed along with dynamical phase in the 'spin-echo' method. If we consider $\theta = \pi/3$ then we get from the antisymmetric Bell state to symmetric entangled state.

$$\Psi(t = \tau) = (\alpha| \uparrow (t) > | \downarrow (t) > + |\beta| \uparrow (t) > | \downarrow (t) > \tag{37}$$

This is analogous to the physics of fermion-boson transmutation. Hence by varying the angle $\theta : 0 \longrightarrow \pi/3 \longrightarrow \pi/2$, we can continuously change Berry phase $\gamma : 0 \longrightarrow \pi/2 \longrightarrow \pi$, of the antisymmetric Bell singlet state Ψ_- to the symmetric Bell state Ψ_+ and back to Ψ_-[14].

From the eqn. 20 we note that for very large $L(L \longrightarrow \infty)$, μ tends to zero. Now from the eqn. 33, we note that the limit $\mu \longrightarrow 0$ is achieved when the angular displacement of the magnetic flux line is such that for up spin $\cos\theta = 1$ and for down spin $\cos\theta = -1$ indicating the value of $\theta = 0$ to π respectively.

Our above analysis help us to argue that Berry phase factor $\gamma_{\uparrow,\downarrow} = \pi(1 \mp \cos\theta)$ can be taken to be a measure of entanglement. Indeed the measure of entanglement is usually taken to be given by the concurrence C. A pure general bipartite state is given by

$$|\psi >= \alpha| \downarrow\downarrow> +\beta| \downarrow\uparrow> +\gamma| \uparrow\downarrow> +\delta| \uparrow\uparrow> \tag{38}$$

where α, β, γ and δ are complex coefficients satisfying the normalization condition. The complex concurrence is defined by [15]

$$C = 2(\alpha\delta - \beta\gamma) \tag{39}$$

So from eqn.(35) we note that the concurrence for the given state(let it be up) is $C = 2\beta\gamma$. Now we can relate the norm of this concurrence with $\mu_\uparrow = \frac{1}{2}(1 - \cos\theta)$, the corresponding Berry phase being $\pi(1 - \cos\theta)$. Indeed when $\theta = 0$ (i.e there is no displacement of the magnetic flux line) we have

$$|C| = 0$$

which means disentanglement for up spin. For $\theta = \pi$ i.e. there is maximum displacement of the magnetic flux line, we have

$$|C| = 1$$

through the value $\mu_\uparrow = 1$. Thus within the range of $0 \leq \mu_\uparrow \leq 1$ the measure of entanglement is associated with the norm of the complex concurrence.

It is noted that when the Berry phase factor $\mu_\uparrow$ vanishes, we will have zero magnetic field indicating that there is maximum disorder. Though disorder and order state depend on temperature, when $\mu_\uparrow$ is maximum, we have an order state. This suggests that we can relate the measure of entanglement with entropy through Berry phase. Indeed there is a relationship between the entropy after entanglement with the concurrence which is given by

$$f(c) = H(1 + \frac{\sqrt(1 - C^2)}{2})$$

where $H(x)$ is the shannon entropy. So substituting $\mu_\uparrow = 0$ (disentanglement) we find $f(c) = H(1)$ and for maximum entangled state $\mu_\uparrow = 1$ yield $f(c) = H(1/2)$. Thus the maximum entangled state for up spin can be realized through max. value of Berry phase when the entropy is minimum indicating a highly correlated state.

4 Noise and Berry phase

Motivated by the works of Chiara and Palma [16] on the influence of classical fluctuation of field on the Berry phase of spin 1/2 particle, we now like to find the effect of noise in the Berry phase of quantized spinor and in its entangled state both in the presence and the absence of 'spin-echo'method. We define noise by a shift in chirality. If we consider that with the lapse of time, the parameter λ suffers a deviation $\lambda \longrightarrow \lambda + \delta\lambda$ due to any change in θ, ϕ and χ resulting a gauge transformation.

$$\mathbf{A}(\lambda) \longrightarrow A(\lambda) + \frac{\partial A(\lambda)}{\partial \lambda}\delta\lambda \tag{40}$$

Here $A(\lambda)$ is the gauge connection associated with the Lagrangian in eqn.7 giving rise to Berry phase. This fluctuation of gauge connections by the parameter λ, is the very cause of shift in magnetic flux line corresponding chiral symmetry breaking. Now from equation 8. considering the spin up case, we have

$$A^\uparrow(\lambda) = \frac{1}{2}(1 - \cos\theta) \tag{41}$$

This leads to have the noise dependent Berry connection of the quantized spinor

$$\mathbf{A}^\uparrow(\lambda) = \frac{1}{2}(1 - \cos\theta + \sin\theta\delta\theta) \tag{42}$$

which results a modification of the Berry phase

$$\Gamma_\uparrow = \pi(1 - \cos\theta + \sin\theta\delta\theta) = \gamma_\uparrow + \triangle\gamma \tag{43}$$

and similarly for down spinor

$$\Gamma_\downarrow = \pi(1 + \cos\theta - \sin\theta\delta\theta) = \gamma_\downarrow - \triangle\gamma \tag{44}$$

where we consider $\Gamma_{\uparrow,\downarrow}$ as the noise induced Berry phase for the spin up and down quantized particles respectively.

For the entangled state of two identical spinor, as we find in equation 25, that the evolution of the state at a particular instant depends on the difference of $\gamma_\uparrow, \gamma_\downarrow$ which implies increase of noise by twice. The effect of noise in the entangled state formed after 'spin-echo' will be less as realized from eqn.36

At the end, we like to comment that here the noise is responsible for the fluctuation of quantization that can be applied for the entanglement of Quantum Hall particles in the plateau and non-plateau region.

Discussions

We here express the quantized spinor in terms of action of quantum gates on two qubits $|0>$ and $|1>$ that represent the ground and excited state respectively. The one qubit rotation of the spinor results it to change from one state to other with the variation of Berry phase. The dynamical phase can be removed in the spin-echo method. In this method the inclusion of Berry phase in the entangled state is responsible for the measure of entanglement. Fluctuation in chirality is considered as noise that modify the fixed value of Berry phase. The effect of noise doubles as two pure identical spinor entangle. We like to study further this effect of noise, decoherence and entanglement in connection with quantization aspect of Berry phase in other quantum systems.

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